Although the requirements for computer time and disk storage are still large, they are orders of magnitude less than what would be required by numerical solution with the differential formulation, so that some time-dependent, three-dimensional solutions can at least be considered within the present state of computer development. The accurate prediction of body forces at high Reynolds numbers would require unfeasible amounts of computer time and storage with a rectangular constant-mesh coordinate system and is still beyond the present development of the technique. Further work would be desirable to extend the present technique to nonrectangular, and eventually general, nonuniform curvilinear coordinate systems, so that bodies of general shape can be more efficiently treated.

References

[1] Thompson, J. F., "Two Approaches to the Three-Dimensional Jet-in-Cross-Wind Problem: A Vortex Lattice Model and a Numerical Solution of the Navier-Stokes Equations," Ph.D. thesis, 1971, Georgia Institute of Technology, Atlanta, Ga.

[2] Wu, J. C. and Thompson, J. F., "Numerical Solution of the Unsteady, Three-Dimensional Navier-Stokes Equations," *Proceedings of Project SQUID Workshop of Fluid Dynamics of Unsteady, Three-Dimensional, and Separated Flows,* Purdue University, Lafayette, Ind., 1971.

[3] Wu, J. C. and Thompson, J. F., "Numerical Solutions of Time-Dependent Incompressible Navier-Stokes Equations Using an Integro-Differential Formulation," *Computers and Fluids,* Vol. 1, 1973, pp. 197–217.

[4] Payne, R. B., "Calculations of Unsteady Viscous Flow Past a Circular Cylinder," *Journal of Fluid Mechanics,* Vol. 4, 1958, pp. 81–86.

[5] Payne, R. B., "A Numerical Method for Calculating the Starting and Perturbation of a Two-Dimensional Jet at Low Reynolds Number," R & M 3047, 1958, Great Britain Aeronautical Research Council, London, England.

[6] Ingham, D. B., "Note on the Numerical Solution for Unsteady Viscous Flow Past a Circular Cylinder," *Journal of Fluid Mechanics,* Vol. 31, 1968, pp. 815–818.

[7] Chorin, A. J., "Numerical Study of Slightly Viscous Flow," *Journal of Fluid Mechanics,* Vol. 57, 1973, pp. 785–796.

[8] Roache, P. J., *Computational Fluid Dynamics,* Hermosa Publishers, Albuquerque, N.Mex., 1972, pp. 9–10.

[9] Karamcheti, K., *Principles of Ideal-Fluid Aerodynamics,* Wiley, New York, 1966, pp. 269–273.

[10] Richtmyer, P. D. and Morton, K. W., *Difference Methods for Initial-Value Problems,* 2nd ed., Interscience, New York, 1967, p. 176.

[11] Shanks, S. P., "An Explicit Numerical Solution of the Three-Dimensional, Incompressible, Time-Dependent Navier-Stokes Equations for a Body with Rectangular Boundaries," M.S. thesis, 1972, Mississippi State University, State University, Miss.

[12] Varga, R. S., *Matrix Iterative Analysis,* Prentice-Hall, Englewood Cliffs, N.J., 1962, pp. 56–59.

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Supersonic, Turbulent Boundary-Layer Separation

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Results are presented of an experimental investigation of turbulent boundary-layer separation in a compression corner in supersonic flow. The experiments were conducted at a freestream Mach number of 2.96 for freestream length Reynolds numbers of 10^7 to 10^8 at adiabatic wall conditions. Surface static pressure measurements, oil flow studies and schlieren and interferometry photographs of the flowfield were made for a series of compression ramp angles so that attached, incipient and well separated turbulent interaction regions could be studied at each freestream condition. The model consisted of a long flat plate followed by an adjustable compression ramp. The turbulent boundary-layer thickness at the compression corner varied between approximately 0.18 in. and 0.12 in. giving a corresponding range of Re_δ of 10^5 to 10^6 for the investigation. The length of the separated region was found to decrease with increasing Reynolds number for fixed compression ramp angle, even when nondimensionalized by the boundary-layer thickness. The incipient separation angle was found to increase with increasing Reynolds number. This result agrees with data obtained by Roshko and Thomke on a wind-tunnel sidewall at higher values of Re_δ.

Nomenclature

C_{f_0} = skin-friction coefficient at beginning of interaction
M = Mach number
p = pressure
$Re_{\infty L}$ = freestream Reynolds number based on flat plate length
Re_δ = Reynolds number based on boundary-layer thickness
T_w = wall temperature
x = axial distance measured along surface from leading edge
α_R = compression ramp angle
δ = boundary-layer thickness

Subscripts
B = beginning of interaction
i = incipient condition
s = condition at the separation point
R = condition at the reattachment point
∞ = freestream condition

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Introduction

MUCH of our knowledge of high-speed turbulent boundary-layer separation is limited to relatively low Reynolds numbers (10^6 to 10^7). The Reynolds numbers associated with flow separation in actual flight, however, are generally higher ($\sim 10^8$). A review of interaction problems of practical interest in high-speed flight and the advances in our understanding of these problems is given in Ref. 1. Only recently have facilities become available to investigate the high Reynolds number range (10^7 to 10^8) on models.[2-6] The only investigation published to date at very high Reynolds numbers was performed by Roshko and Thomke[7] on a wind-tunnel wall with an Re_δ of 10^6 to 10^7 corresponding to an effective length Reynolds number of 10^8 to 10^9.

From the existing body of information, turbulent boundary-layer separation appears to be relatively insensitive to Reynolds number and wall temperature, but depends significantly on Mach number.[1] Investigations at relatively low Reynolds numbers (10^6 to 10^7)[8-11,16] have shown a decrease in incipient separation angle with increasing Reynolds number for all Mach numbers. The same trend has also been reported for hypersonic Mach numbers at Reynolds numbers somewhat greater than 10^7.[2-6] The opposite trend, that of increasing incipient separation angle with increasing Reynolds number, was observed by Roshko and Thomke[7] at very high Reynolds numbers.

The observance of a reversal in the trend of incipient separation angle with Reynolds number may be an important feature of the turbulent viscous-inviscid interaction. Todisco and Reeves,[12] and later Elfstrom,[3] postulate that the reversal in incipient separation angle with Reynolds number closely follows the development of the wake component in the velocity profile. The analysis of Elfstrom[3] shows that the trend reversal occurs at increasing values of Re_δ for increasing Mach number. Holden[4] points out that experimental techniques may account for the observed differences in α_i trend with Re_δ, and that model geometry, tunnel geometry, boundary-layer nonequilibrium effects, and tripping could influence measurements.

The present investigation was designed to provide turbulent separation data in the Reynolds number range of 10^7 to 10^8 (Re_δ's of 10^5 to 10^6) by means of a long flat plate in a high unit Reynolds number wind tunnel at a Mach number of 3. Precautions were also taken to render three-dimensional effects negligible.

Experimental Procedure

Wind Tunnel

The experiments were conducted in the Aerospace Research Labs' 8 in. by 8 in. Mach 3 high Reynolds number wind tunnel. This is a blowdown wind tunnel which has a closed jet and operates at stagnation temperature near atmospheric and stagnation pressures of from 50 to 570 psia. Nominal running times are 30 sec to several minutes depending on the stagnation pressure desired. Freestream unit Reynolds numbers of 0.8 to 8.0×10^6 per in. are obtainable in this facility.

Model

The model consisted of a 12 in. long flat plate followed by a 2.5 in. compression ramp whose angle could be varied continuously from 0° to 35° by remote control during a run. Special care was taken to design a leak-tight hinge line with a maximum gap width of less than 0.005 in. The model spanned the wind-tunnel test section and was sealed at the sidewalls. The flat plate leading edge was sharp with a radius of no more than 0.001 in.

Centerline surface pressure taps were distributed longitudinally within a band ±0.375 in. of the centerline. Ninety two pressure taps were used to give data every 0.05 in. in the interaction region. Nine pressure taps were located off-centerline in the interaction region to identify possible three-dimensional effects. Thermocouples mounted near the model surface were used to determine when adiabatic wall conditions existed.

Optical Measurements

Shadowgraph and schlieren photographs and dual holographic interferograms were obtained for each ramp angle and freestream condition investigated. Boundary-layer measurements, shock locations, and density profiles were measured optically thus eliminating the disadvantage of a probe which could disturb the small boundary layer and separated region. The optical techniques and analysis procedures are described in Ref. 13.

Oil Flow Measurements

Silicone oil, with no additives, was used to obtain the oil flow photographs. Variations of the lighting angle and oil viscosity were used to obtain the best photographs of the oil accumulation at the separation point while minimizing the effects of the oil on the interaction.

Separation Point Determination

Four procedures were used to determine the location of boundary-layer separation. The first method assumed that separation occurs at the first inflection point in the surface static pressure distribution in the axial direction. Second, separation was assumed to be located at a specific axial location for the compression ramp angle where there is an inflection point in the static pressure vs ramp angle curve. Third, separation was assumed to be located at the oil accumulation line on the oil flow photographs. The fourth method used to determine the separation point was the shadowgraph and/or schlieren photographs.

Most investigators have found good agreement between separation locations determined from shadowgraphs and schlieren photographs and separation locations determined from static pressure data and oil flow data. However, some investigators have found discrepancies between these techniques and have expressed preferred methods for detecting separation.[4,11,14]

Results and Discussion

A ramp angle variation of from 0° to 26° was sufficient to investigate fully attached and well separated boundary-layer flow at a Mach number of 3 over the range of freestream length Reynolds numbers of 10^7 to 10^8. The boundary-layer thickness at the hinge line varied from approximately 0.12 to 0.18 in. giving a corresponding range in Re_δ of 10^5 to 10^6.

The large ratio of model span to boundary-layer thickness (~ 50) coupled with preliminary oil flow visualization and hinge line and sidewall leak tests gave reasonable assurances that three-dimensional effects were negligible. For the largest ramp angles, reattachment occurred at least 10 boundary-layer

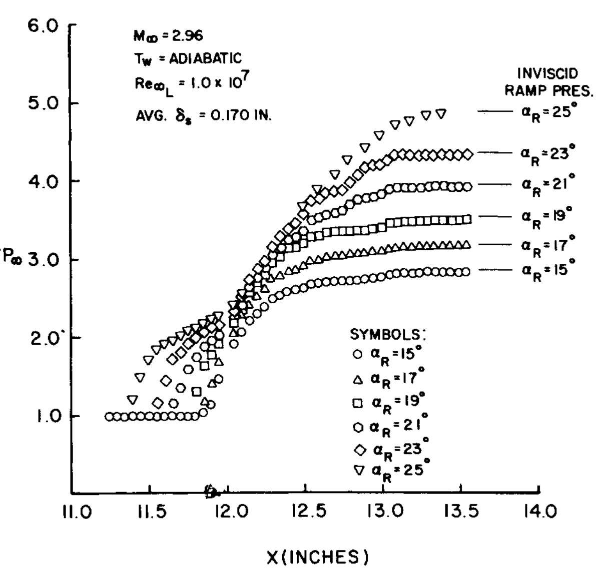

Fig. 1 Surface static pressure distributions as a function of compression ramp angle.

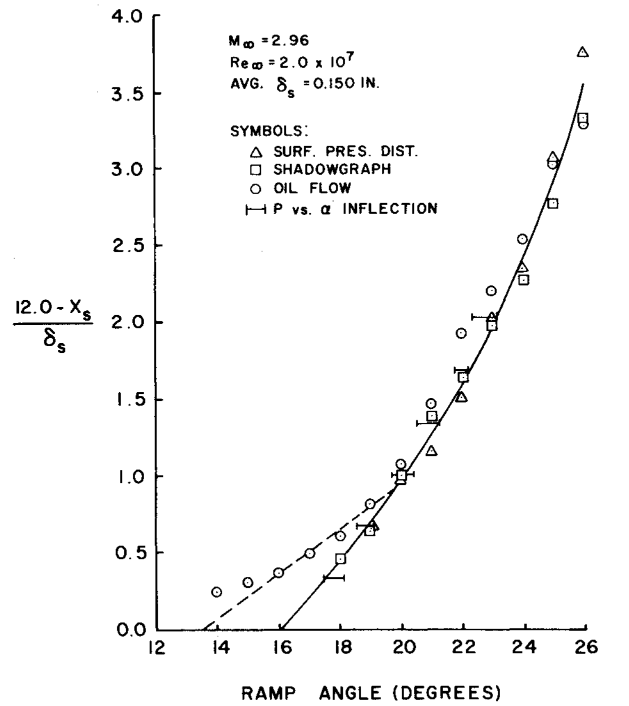

Fig. 2 Variation of separation distance with compression ramp angle.

thicknesses upstream of the ramp trailing edge so that the influence of this trailing edge expansion on the separation process could be expected to be negligible.[15] Excellent agreement was also obtained between the calculated inviscid ramp surface pressure and the measured ramp aft surface pressure.

The surface static pressure distributions for one freestream Reynolds number and several compression ramp angles are shown in Fig. 1. Additional data were obtained for freestream length Reynolds number values of 2×10^7, 5×10^7, and 9×10^7 and compression ramp angles from 10° to 26° in one-degree increments. Variations of surface pressure at specific

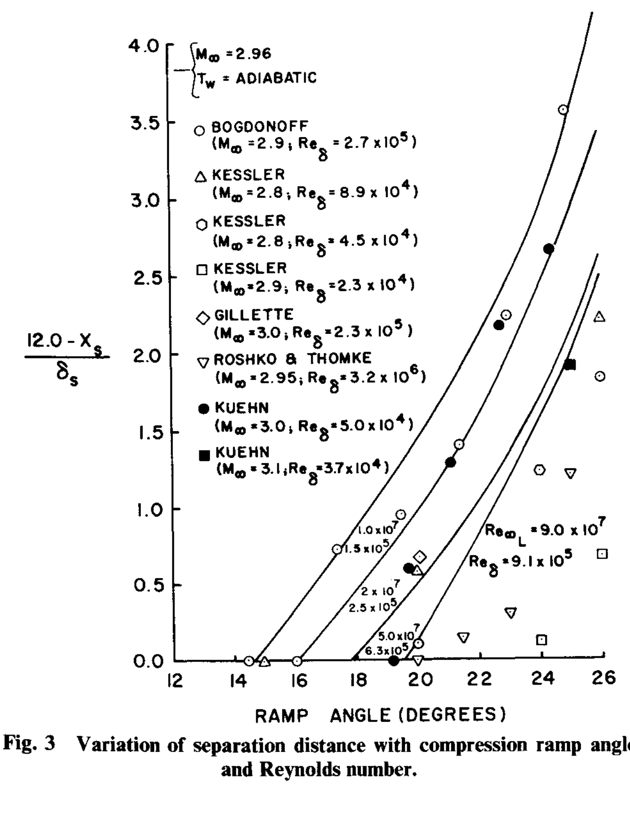

Fig. 3 Variation of separation distance with compression ramp angle and Reynolds number.

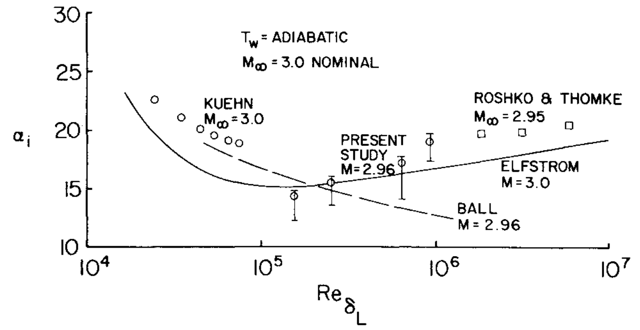

Fig. 4 Comparison of incipient separation data for turbulent interactions with theory.

static pressure orifices with compression ramp angles were also obtained.

Figure 2 presents the distance to separation as a function of compression ramp angle for one Reynolds number as determined by the four methods discussed. Good agreement is shown between these methods and only for small compression ramp angles near incipient does the oil flow data differ from the indicated trend. Figure 3 presents the accumulated data for the four Reynolds numbers investigated. The curves were determined to be the best smooth fits to the data and were extrapolated to zero separation to determine the incipient separation angle.

With the separation distance nondimensionalized by the undisturbed boundary-layer thickness just upstream of separation, the separation distance increases with decreasing Reynolds number for fixed compression ramp angle and the incipient separation angle increases with increasing Reynolds number. Limited data obtained by other investigators at the same nominal Mach number are also shown in Fig. 3.

The boundary-layer separation angle appeared to be relatively constant at approximately 11° for compression ramp angles greater than incipient. The separation angles were obtained from the schlieren photographs by calculating the deflection angles necessary to achieve the observed separation shock angles. These separation angles agreed with those obtained by drawing a straight line from the separation point to the reattachment point obtained from the surface static pressure data as the first and third inflection points in the pressure distributions. However, no better than 25% accuracy could be achieved in locating the third inflection point (reattachment point) from the static pressure distributions (like those shown in Fig. 4). This uncertainty resulted in a ±1.5° variation of the separation angle but no consistent variation with Reynolds number or compression ramp angle was observed. The length of separation (distance from the separation point to reattachment point) could best be obtained for a particular Reynolds number and ramp angle by calculating the length of the line drawn from the separation point (obtained from Fig. 3) to the compression ramp surface at an 11° angle to the flat plate.

A comparison was made with the incipient separation correlation of pressure rise to separation with $C_{f_o} M^3$ by Holden[4] for both shock and wedge induced interactions. The present results were found to be in good agreement with existing data but the reverse trend of increasing incipient separation angle with increasing Reynolds number obtained in the present study and by Roshko and Thomke[7] clearly departs from the trend indicated by the correlation. A comparison is also made with the predictions of Ball[15] and Elfstrom[3] in Fig. 4. Ball's prediction is based on an extension of laminar theory and the assumption that the dependence on Reynolds number of the extent of separation enters only through the incipient separation compression angle. For small values of Reynolds number good agreement is indicated, but for large values of Reynolds number significant departure from experiment exists. This theory does not indicate the existence of a reversal in α_i trend with Re_δ. The method used by Elfstrom[3] is essentially the same as the method of

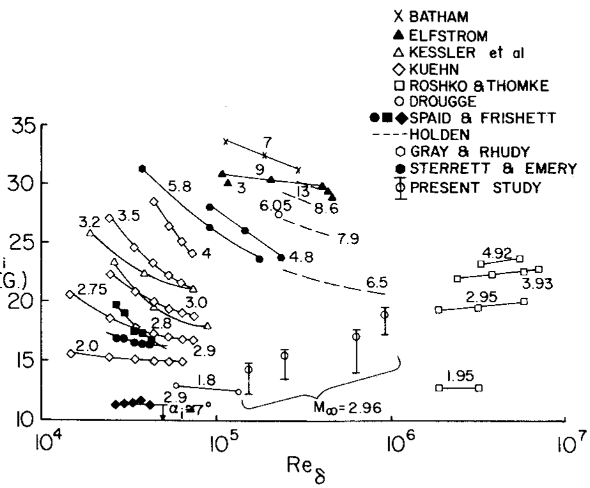

Fig. 5 Comparison of experimentally determined incipient separation angle data for turbulent interactions.

rotational characteristics except that an analytic velocity profile family is used. Here a reversal in α_i trend with Re_δ is indicated and closely follows the development of the wake component of the velocity profile in the calculation. Reasonably good agreement with experiment is shown in magnitude and trend.

The experimental results for the incipient separation angles as determined from the present study are shown in Figs. 4 and 5. The data points are presented with a vertical uncertainty band. The circles were obtained by extrapolating the best curves through the separation distance data points to zero separation distance. The uncertainty bands about these circles include the uncertainty in selecting the proper "best" curves through the data and the departure of the oil flow data from the "best" curves for small separation distances.

A comparison of the results of the present study with existing experimental data is made in Fig. 5. Only the present results and those of Roshko and Thomke,[7] obtained at higher values of Re_δ on a wind-tunnel sidewall, exhibit an increasing value of α_i with increasing Re_δ.

Conclusions

The extent of turbulent boundary-layer separation ahead of a compression corner at Mach 3 has been determined under adiabatic wall conditions and over a range of Re_δ from 10^5 to 10^6. The important conclusions drawn from this investigation were:

1) The separation distance, when nondimensionalized by the boundary-layer thickness, decreases with increasing Re_δ for fixed compression ramp angle.

2) The incipient separation angle increases with increasing Re_δ. This result agrees with that obtained by Roshko and Thomke[7] on a wind-tunnel sidewall at the same nominal Mach number but much higher values of Re_δ.

3) The present results, when compared with those of Kuehn,[8] suggest a reversal in the α_i trend with Re_δ. This trend closely follows the prediction made by Elfstrom[3] suggesting that the reversal follows the development of the wake component of the turbulent boundary-layer velocity profile for high values of Reynolds number whereas for low values of Reynolds number the wall layer of the boundary-layer dominates the interaction.

References

[1] Korkegi, R. H., "Survey of Viscous Interactions Associated with High Mach Number Flight," *AIAA Journal*, Vol. 9, No. 5, May 1971, pp. 771–784.

[2] Batham, J. P., "An Experimental Study of Turbulent Separating and Reattaching Flows at a High Mach Number," *Journal of Fluid Mechanics*, Vol. 52, Pt. 3, 1972, pp. 425–435.

[3] Elfstrom, G. M., "Turbulent Separation in Hypersonic Flow," Imperial College Aero. Rept. 71-16, 1971, University of London, London, England.

[4] Holden, M. S., "Shock Wave-Turbulent Boundary Layer Interaction in Hypersonic Flow," AIAA Paper 72-74, San Diego, Calif., 1972.

[5] Gray, J. D. and Rhudy, R. W., "Investigation of Flat-Plate Aspect Ratio Effects on Ramp-Induced, Adiabatic, Boundary Layer Separation at Supersonic and Hypersonic Speeds," AEDC-TR-70 (235), 1971, Arnold Engineering Development Center, Tullahoma, Tenn.

[6] Sterret, J. R. and Emery, J. C., "Experimental Separation Studies for Two-Dimensional Wedges and Curved Surfaces at $M = 4.8$ to 6.2," TN D-1014, 1962, NASA.

[7] Thomke, G. J. and Roshko, A., "Incipient Separation of a Turbulent Boundary Layer at High Reynolds Number in Two-Dimensional Supersonic Flow over a Compression Corner," DAC 59819, 1969, McDonnell Douglas Astro Co., Western Division, Huntington Beach, Calif.

[8] Kuehn, D. M., "Experimental Investigation of the Pressure Rise Required for the Incipient Separation of Turbulent Boundary Layers in Two-Dimensional Supersonic Flow," Memo 1-21-59A, 1959, NASA.

[9] Kessler, W. C., Reilly, J. F., and Mockayetris, L. J., "Supersonic Turbulent Boundary Layer Interaction with an Expansion Ramp and a Compression Corner," MDCEO264, 1970, McDonnell Douglas, St. Louis, Mo.

[10] Drougge, G., "An Experimental Investigation of the Influence of Strong Adverse Pressure Gradients on Turbulent Boundary Layer at Supersonic Speeds," FFA Rept. 47, 1953, Stockholm, Sweden.

[11] Spaid, F. W. and Frishett, J. C., "Incipient Separation of a Supersonic Turbulent Boundary Layer, Including Effects of Heat Transfer," *AIAA Journal*, Vol. 10, No. 7, July 1972, pp. 915–922.

[12] Todisco, A. and Reeves, B. L., "Turbulent Boundary Layer Separation and Reattachment at Supersonic and Hypersonic Speeds," *Proceedings of the Symposium on Viscous Interaction Phenomena in Supersonic and Hypersonic Flow*, AD 714 362, Hypersonic Research Lab., Aerospace Research Labs., Wright-Patterson Air Force Base, Ohio, 1969.

[13] Havener, A. G. and Radley, R. J., Jr., "Turbulent Boundary-Layer Separation Measurements Using Holographic Interferometry," AIAA Paper 73-664, Palm Springs, Calif., 1973.

[14] Bogdonoff, S. M., "Some Experimental Studies of the Separation of Supersonic Turbulent Boundary Layers," *Proceedings of the Heat Transfer and Fluid Mechanics Inst.*, Univ. of California, Los Angeles, Calif., June 1955, Sec. V., pp. 1–23.

[15] Ball, K. O. W., "A Summary of the Factors Influencing the Extent of Separation of a Laminar Boundary Layer Due to a Compression Corner of Moderately Hypersonic Speeds," ARL 70-0065, 1971, Aerospace Research Labs., Wright-Patterson Air Force Base, Ohio.

[16] Gillette, W. B., "Separation Measurements of Supersonic Turbulent Boundary Layers over Compression Corners," AD 644-194, 1966, Univ. of Texas, Austin, Texas.